

Optimizing inhomogeneous spin ensembles for quantum memory

Guy Bensky,¹ David Petrosyan,² Johannes Majer,³ Jörg Schmiedmayer,³ and Gershon Kurizki¹

¹*Department of Chemical Physics, Weizmann Institute of Science, Rehovot 76100, Israel*

²*Institute of Electronic Structure and Laser, FORTH, GR-71110 Heraklion, Crete, Greece*

³*Vienna Center for Quantum Science and Technology (VCQ),
Atominstytut, TU-Wien, Stadionalle 2, 1020, Vienna, Austria*

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We propose a method to maximize the fidelity of quantum memory implemented by a spectrally inhomogeneous spin ensemble. The method is based on preselecting the optimal spectral portion of the ensemble by judiciously designed pulses. This leads to significant improvement of the transfer and storage of quantum information encoded in the microwave or optical field.

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I. INTRODUCTION

Recent experimental demonstrations of strong coupling between spin ensembles (SEs) and microwave photons of superconducting resonators [1–6] are an important step towards realizing functional, hybrid quantum devices [7–18]. Such hybrid devices may benefit from combining the advantageous properties of very different subsystems, or “blocks”: (i) a quantum processor block containing, e.g., quantum dots [19, 20] or superconducting qubits [21–24] which can perform rapid quantum gate operations but are vulnerable to decoherence due to their strong coupling to the environment and/or the noise of the external controls; (ii) a quantum memory block consisting of a SE of active dopants in a solid [1–6, 25, 26] or trapped ultracold atoms [7–15] which are weakly coupled to the environment and therefore are suitable for information storage; (iii) a quantum “bus” or interface, such as a microwave cavity, whose interaction with the other blocks can be quickly switched on and off by, e.g., tuning in and out of resonance [27–30]. A related scenario concerns reversible transfer and storage of optical excitations in SEs used as memories for photonic quantum repeaters [31–37].

The potential advantages of hybrid quantum devices are countered by decoherence during the transfer and storage of quantum information (QI), which is rooted in the homogeneous (lifetime) broadening and the inhomogeneous spectral width of the SE constituting the memory block. Using magnetic dipole or optical Raman transitions can greatly prolong the relaxation time of the spins. But since typically the coupling of individual spins with the microwave cavity or optical field is weak, QI must be collectively encoded in an ensemble of $N \gg 1$ spins, which are prone to spectral broadening [18, 38, 39]. This broadening introduces dephasing on the memory block which can both hinder the transfer of QI and limit its coherent storage time. Although conventional spin-echo techniques [40–43] can prolong the storage time, they may be impractical for spectrally broad SEs, and they cannot improve the QI transfer (in fact they would preclude the transfer altogether).

Here, after analyzing the fidelity of SE quantum memory, we present a method that can significantly improve its performance. The method is based on preselecting the optimal spectral portion of the inhomogeneously broadened ensemble using judiciously designed pulse(s), followed by the transfer of all the spurious spins to an auxiliary metastable state. Although the selected subensemble contains far fewer spins and therefore requires longer QI transfer times, we find that the resulting fidelity of the quantum memory during the QI transfer and storage is much higher under experimentally realistic conditions.

II. FIDELITY OF AN ENSEMBLE QUANTUM MEMORY

To set the stage for the discussion, assume that a qubit is encoded in a SE as a superposition of its collective ground state $|\psi_0\rangle = |g_1, g_2, \dots, g_N\rangle$ and the fully-symmetrized single-excitation (Dicke) state $|\psi_1\rangle = N^{-1/2} \sum_j |j\rangle$, where $|j\rangle = |g_1, g_2, \dots, e_j, \dots, g_N\rangle$ denotes a state with only spin j excited. If all the spins had the same resonant frequency ω_0 on the transition $|g\rangle \rightarrow |e\rangle$, the single-excitation state would evolve in time $\tau > 0$ as $|\psi_1(\tau)\rangle = N^{-1/2} \sum_j e^{-i\omega_0\tau} |j\rangle$, while the ground state $|\psi_0\rangle$ remains unchanged. Yet, due to inhomogeneous broadening, each excited spin has different resonant frequency ω_j . Then, even if the symmetric state $|\psi_1\rangle$ is prepared at $\tau = 0$, it would evolve into $|\tilde{\psi}_1(\tau)\rangle = N^{-1/2} \sum_j e^{-i\omega_j\tau} |j\rangle$. We may thus define the storage fidelity at time $\tau \geq 0$ as the squared overlap of state $|\psi_1(\tau)\rangle$ with its inhomogeneously-broadened counterpart $|\tilde{\psi}_1(\tau)\rangle$:

$$F(\tau) \equiv |\langle \psi_1(\tau) | \tilde{\psi}_1(\tau) \rangle|^2 = \left| \frac{1}{N} \int n(\omega) e^{-i(\omega - \omega_0)\tau} d\omega \right|^2, \quad (1)$$

where $n(\omega)$ is the ensemble spectral density normalized to the total number of spins, $\int n(\omega) d\omega = N$. Note that $F(0) = 1$ for any $n(\omega)$. Since the ground state $|\psi_0\rangle$ does not evolve in time, $F(\tau)$ quantifies how the fidelity of information *already encoded* in a SE decreases over time.

Specifically, for an ensemble with Lorentzian spectrum of width Δ , $n(\omega) = n_0[1 + (\omega - \omega_0)^2/\Delta^2]^{-1}$, the fidelity loss is exponential in time, $F(\tau) = e^{-2\Delta\tau}$. As shown below, Eq. (1) also characterizes the efficiency of excitation transfer to and from the SE.

A. Transfer fidelity

Consider a single-mode field of a microwave cavity or an optical beam interacting with the SE. Assume that initially the field contains a single excitation (photon) of frequency ω_0 and the SE is in the ground state $|\psi_0\rangle$. Each spin j in the ensemble interacts with the field on the transition $|g_j\rangle \rightarrow |e_j\rangle$ with the coupling strength η_j . In the rotating wave approximation, neglecting the field and spin relaxations, their combined state at any time τ can be written as $|\Psi(\tau)\rangle = \alpha(\tau)|1, \psi_0\rangle + \sum_j \beta_j(\tau)e^{-i(\omega_j - \omega_0)\tau}|0, j\rangle$, where $|1, \psi_0\rangle$ refers to the state with a single photon and all the spins in the ground state, and $|0, j\rangle$ denotes the field vacuum and the j th spin excited. The probability amplitudes α and β_j evolve in time according to

$$\begin{aligned}\dot{\alpha} &= -i \sum_j \eta_j^* \beta_j e^{-i(\omega_j - \omega_0)\tau}, \\ \dot{\beta}_j &= -i \eta_j \alpha e^{i(\omega_j - \omega_0)\tau},\end{aligned}$$

which yields $\dot{\alpha}(\tau) = -\sum_j |\eta_j|^2 \int_0^\tau d\tau' \alpha(\tau') e^{-i(\omega_j - \omega_0)(\tau - \tau')}$. Assuming that the spin-field couplings η_j are not correlated with the transition frequencies ω_j [3–5], we finally obtain

$$\dot{\alpha}(\tau) = -\bar{\eta}^2 N \int_0^\tau d\tau' \alpha(\tau') \sqrt{F(\tau - \tau')}, \quad (2)$$

where $\bar{\eta}^2 \equiv N^{-1} \sum_i |\eta_i|^2$ and $F(\tau)$ is given by Eq. (1). With $F(\tau) = 1$ for all τ , Eq. (2) predicts Rabi oscillations between the field and the SE according to $\alpha(\tau) = \cos(\bar{\eta}\sqrt{N}\tau)$, so that at time $\tau_{\text{tr}} \equiv \pi/\bar{\eta}\sqrt{N}$ there is a full retrieval of the excitation into the field, $|\alpha(\tau_{\text{tr}})|^2 = 1$ [with the combined state $|\Psi(\tau_{\text{tr}})\rangle = -|1, \psi_0\rangle$ acquiring a π phase shift]. In the presence of inhomogeneous broadening, however, $F(\tau)$ decreases with time, resulting in damped Rabi oscillations. The fidelity of the transfer followed by retrieval is the value of $|\alpha(\tau_{\text{tr}})|^2$ after one such Rabi oscillation. For $F(\tau) \lesssim 1$ the fidelity loss is $\mathcal{O}(1 - F)$. Hence, increasing the storage fidelity $F(\tau)$ for all times $\tau \in [0, \tau_{\text{tr}}]$ also improves the transfer fidelity.

B. The optimal spectrum

Our goal is to filter out of the SE with broad spectrum $n(\omega)$ a subensemble with the spectrum $n'(\omega)$ that will maximize the resulting fidelity F' while still containing many spins $N' = \int n'(\omega)d\omega \gg 1$. Before discussing the filtering procedure, let us deduce the optimal spectrum

of the subensemble. To this end, we may consider two different tasks: (a) QI transfer to and from the memory, and (b) QI storage in the memory for a specific time τ_s .

(a) For QI transfer, we require that the fidelity $F'(\tau)$ for the selected subensemble be high for all $\tau \in [0, \tau_{\text{tr}}]$. For any symmetric spectrum $n'(\omega)$, Eq. (1) can be expanded in a Taylor series

$$F'(\tau) = \left| \sum_{k=0}^{\infty} \frac{(-1)^k \tau^{2k}}{(2k)!} \langle (\omega - \omega_0)^{2k} \rangle \right|^2 \approx 1 - \langle (\omega - \omega_0)^2 \rangle \tau^2, \quad (3)$$

where $\langle (\dots) \rangle \equiv \int n'(\omega)(\dots)d\omega / \int n'(\omega)d\omega$ denotes the average over the spectral distribution $n'(\omega)$. Hence, the spectral variance $\langle (\omega - \omega_0)^2 \rangle$, which is the leading term of the expansion, should be as small as possible for a required number N' of the selected spins. We then find that the optimal spectrum is $n'(\omega) = n(\omega)$ for $|\omega - \omega_0| < \Delta$ and $n'(\omega) = 0$ otherwise, where Δ is such that $N' = \int_{\omega_0 - \Delta}^{\omega_0 + \Delta} n(\omega)d\omega$. In other words, we should select all the spins from the frequency interval $\omega \in [\omega_0 - \Delta, \omega_0 + \Delta]$, and none outside of it. Assuming an approximately constant original spectrum $n(\omega) \simeq n_0$ around $\omega = \omega_0$, we obtain $N' \simeq 2\Delta n_0$ and

$$F'(\tau) \simeq \text{sinc}^2(\Delta\tau) \simeq 1 - \frac{1}{3}\Delta^2\tau^2 \quad (\Delta\tau \ll 1). \quad (4)$$

(b) For QI storage over time τ_s , we require that the fidelity $F'(\tau)$ be high at exactly $\tau = \tau_s$. For a symmetric spectrum $n'(\omega)$, the fidelity of Eq. (1) reads

$$F'(\tau_s) = \left| \frac{1}{N'} \int n'(\omega) \cos[(\omega - \omega_0)\tau_s] d\omega \right|^2. \quad (5)$$

Hence, we should select all the spins with frequencies ω which maximize $\cos[(\omega - \omega_0)\tau_s]$, i.e., a comb-like spectrum $n'(\omega)$ peaked at $\omega \simeq \omega_k = \omega_0 + 2\pi k/\tau_s$, where $k = 0, \pm 1, \pm 2, \dots$. For a required number of selected spins N' , the optimal spectrum is then composed of a series of rectangular distributions around frequencies ω_k : $n'(\omega) = n(\omega)$ for $|\omega - \omega_k| < \Delta$ and $n'(\omega) = 0$ otherwise (we note a similar result in [37]). Assuming the original spectrum changes little around each $\omega = \omega_k$, $n(\omega) \simeq n_k$, we obtain $N' \simeq 2\Delta \sum_k n_k$ and

$$F'(\tau_s) \approx \text{sinc}^2(\Delta\tau_s) \simeq 1 - \frac{1}{3}\Delta^2\tau_s^2 \quad (\Delta\tau_s \ll 1), \quad (6)$$

which has the same form as Eq. (4), but with important differences. First, the memory now rephases only at integer multiples of τ_s [32, 33, 37]. Second, the wider is the original spectrum $n(\omega)$, the larger number of peaks at ω_k with $n_k > 0$ we can select. For a fixed number of spins N' , this allows for narrower Δ , leading to higher fidelity at τ_s . Alternatively, for the same fidelity (fixed Δ), this results in a larger number of spins N' .

III. FILTERING OF THE ENSEMBLE

We now present the filtering procedure in an ensemble of N active dopants, while its optimization is the subject

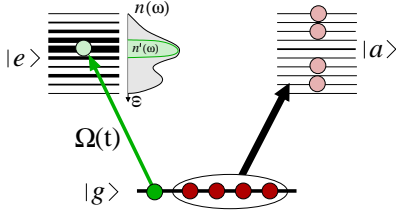


FIG. 1. Level scheme of dopants: $|g\rangle$ and $|e\rangle$ are the ground and excited states of the spin- $\frac{1}{2}$ subspace, and $|a\rangle$ is the auxiliary metastable state. The subensemble with desired spectrum $n'(\omega)$ is selected by the $\Omega(t)$ field, which is followed by transferring all the remaining dopants to $|a\rangle$.

of the following Section.

To select a subensemble of spins with spectrum $n'(\omega)$, we employ another (auxiliary) long-lived state $|a\rangle$ outside the spin- $\frac{1}{2}$ subspace $\{|g\rangle, |e\rangle\}$ of the dopants (see Fig. 1). The preparation of the subensemble proceeds in three steps: (i) Starting with all the dopants in the ground state $|g\rangle$, apply an external pulse of Rabi frequency $\Omega(t)$ that excites them to state $|e\rangle$. The duration T of the pulse should be long enough, in order to select only the dopants with transition frequencies ω within a range of $\Delta \sim 2\pi/T$ around the desired frequency ω_0 , while the shape of $\Omega(t)$ is designed to optimize the resulting frequency spectrum. (ii) Transfer all the dopants remaining in $|g\rangle$ to the auxiliary state $|a\rangle$ by another strong field, using an adiabatic sweep across the $|g\rangle \rightarrow |a\rangle$ transition [41, 42]. (iii) Return the dopants selected in step (i) from $|e\rangle$ to $|g\rangle$ by, e.g., the adiabatic transfer.

The chosen subensemble is now ready to use. Its spectrum is given by $n'(\omega) = n(\omega)P(\omega - \omega_0)$, where

$$P(\omega - \omega_0) = \left| \langle e | T_+ e^{-i \int_0^T H(t) dt} | g \rangle \right|^2 \quad (7)$$

is the probability to excite the spin with transition frequency ω by the preparation pulse $\Omega(t)$ used in step (i). We consider only the amplitude-modulated field $\Omega(t)$ with the fixed carrier frequency ω_0 ; the corresponding Hamiltonian is $H(t) = \frac{1}{2}(\omega - \omega_0)\sigma_z + \Omega(t)\sigma_x$ with $\sigma_{z,x}$ the Pauli spin operators. With the new spectrum $n'(\omega)$, the memory fidelity is given by

$$\sqrt{F'(\tau)} = \frac{1}{N} \int n(\omega) P(\omega - \omega_0) e^{-i(\omega - \omega_0)\tau} d\omega. \quad (8)$$

Ideally, we would like all the spins at the resonant frequency $\omega = \omega_0$ to be selected, $P(0) = 1$, setting the pulse area $A \equiv \int \Omega(t) dt = \pi/2$ (a π -pulse). This still leaves us the freedom to choose the shape of $\Omega(t)$ so as to maximize the fidelity in Eq. (8).

As an example, consider a square preparation pulse $\Omega(t) = \pi/2T$ ($t \in [0, T]$), for which Eq. (7) can be solved exactly, $P(\omega - \omega_0) = \frac{\pi^2}{4} \text{sinc}^2(\frac{1}{2}\sqrt{\pi^2 + (\omega - \omega_0)^2 T^2})$. Using Eq. (1), we find linear fidelity loss $F'(\tau) \approx 1 - 4\tau/T$ for short times $\tau \ll T$. With such a “naïve” choice of $\Omega(t)$, the spectral variance $\langle (\omega - \omega_0)^2 \rangle$ does not converge due to the

long wings of $P(\omega - \omega_0)$ (see Fig. 2), leading to poor fidelity (Fig. 3).

IV. OPTIMIZING THE PREPARATION

The ideal rectangular spectrum of subensemble found in Sec. II B would require infinitely long preparation time. Our goal is therefore to find the optimal preparation pulse $\Omega(t)$ of total duration T which should be shorter than the spin relaxation time. To this end, we employ the method of Lagrange multipliers to maximize the fidelity $F'(\tau)$ for a given number of selected spins N' . [Recall that N' determines the transfer time $\tau_{\text{tr}} \propto (N')^{-1/2}$, which in turn should be smaller than the decay time of the (cavity) field.] For convenience, we will actually maximize $N'\sqrt{F'}$ using the number spins N' and the pulse area A as the constraints. The resulting Euler-Lagrange equation reads

$$\frac{\partial [N'\sqrt{F'(\tau)}]}{\partial \Omega(t)} = \lambda_1 \frac{\partial N'}{\partial \Omega(t)} + \lambda_2 \frac{\partial A}{\partial \Omega(t)}, \quad (9)$$

where λ_1 and λ_2 are the Lagrange multipliers. We will pursue solutions of this equation yielding the optimal preparation pulse $\Omega(t)$.

A. Approximate analytic solutions

Although Eq. (9) can be studied numerically, it is instructive to solve an approximate version of this equation analytically.

From Eq. (7), in second order in Ω , we have

$$P(\omega) \approx \left| \int_0^T \Omega(t) e^{-i(\omega - \omega_0)t} dt \right|^2. \quad (10)$$

Assuming that $P(\omega)$ is much narrower than the initial spectral distribution $n(\omega)$, we obtain

$$F'(\tau) \approx \frac{4\pi^2 n_0^2}{N'^2} \left| \int_0^T \Omega(t + \tau) \Omega(t) dt \right|^2, \quad (11)$$

with $N' \approx 2\pi n_0 \int_0^T \Omega^2(t) dt$ and $n_0 = n(\omega_0)$. Note that for $\tau > T$ the fidelity vanishes, i.e., one cannot store QI for time τ longer than the preparation time T . Equation (9) now reduces to

$$\Omega(t + \tau) + \Omega(t - \tau) = \lambda_1 \Omega(t) + \lambda_2, \quad (12)$$

where we used the constraint $A = \int_0^T \Omega(t) dt$.

Clearly, near the resonance $|\omega - \omega_0| \lesssim \max[\Omega(t)]$ Eq. (10) is incorrect as the selection probability $P(\omega \simeq \omega_0) \lesssim \frac{\pi^2}{4}$ becomes larger than 1 [see Fig. 2(b)]. This leads to the overestimate of the number of selected spins N' , making the corresponding constraint inexact. We will see

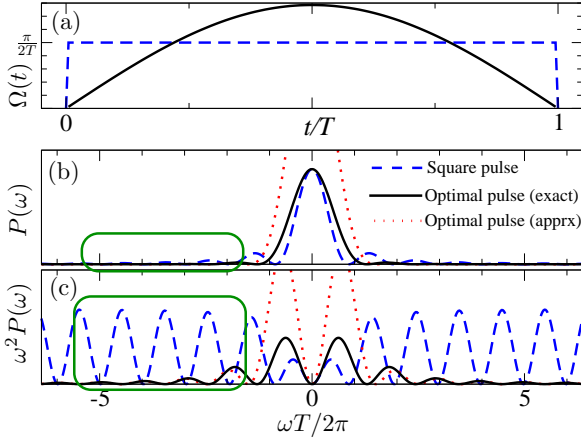


FIG. 2. (a) Square (blue dashed line) and optimal (black solid line) preparation pulses. (b) The corresponding selection spectra $P(\omega)$ calculated numerically. The approximate selection spectrum of Eq. (10) for the optimal pulse is also shown (red dotted line); the exact and approximate spectra differ significantly around resonance but start to coincide for $\omega T/2\pi \gtrsim 1$. (c) Second moments $\omega^2 P(\omega)$ for the corresponding selection spectra.

below, however, that the maximal fidelity obtained under this approximation is close to the exact, numerically calculated fidelity, especially for $\tau \ll T$ [Fig. 3]. This is because the loss of fidelity is mainly due the wings of the selected spectrum, where the behavior of Eq. (10) is correct. We also note that moderate nonuniformity of preparation pulse $\Omega(t)$ for different spins would decrease $P(\omega \simeq \omega_0)$ and thereby the number of selected spins N' , but it would affect little the wings of the selected spectrum $P(\omega)$ and the resulting fidelity F' .

We now seek the optimal preparation pulse which will maximize the fidelity (a) over a continuous time interval $\tau \in [0, \tau_{\text{tr}}]$ (for QI transfer), and (b) at a specific time τ_s (for QI storage).

(a) To maximize the fidelity for all times $\tau \leq \tau_{\text{tr}} \ll T$, we notice that Eq. (12) becomes independent of τ :

$$\ddot{\Omega}(t) = -\tilde{\lambda}_1 \Omega(t) + \tilde{\lambda}_2, \quad (13)$$

where $\tilde{\lambda}_1 = -(\lambda_1 - 2)/\tau^2$ and $\tilde{\lambda}_2 = \lambda_2/\tau^2$ are the rescaled Lagrange multipliers. The highest fidelity is then achieved with the pulse

$$\Omega(t) = \Omega_0 \sin(\pi t/T), \quad (14)$$

where $\Omega_0 = \frac{\pi^2}{4T}$, so that $A = \pi/2$ and $N' \approx \pi^5 n_0 / 16T$. The optimal pulse (14) and the corresponding (exact and approximate) selection spectrum $P(\omega)$ and its second moment $\omega^2 P(\omega)$ are shown in Fig. 2. Due to the suppressed wings of $P(\omega)$, the spectral variance converges to $\langle (\omega - \omega_0)^2 \rangle \sim \pi^2/T^2$, while it does not converge for the square preparation pulse. The resulting fidelity is shown

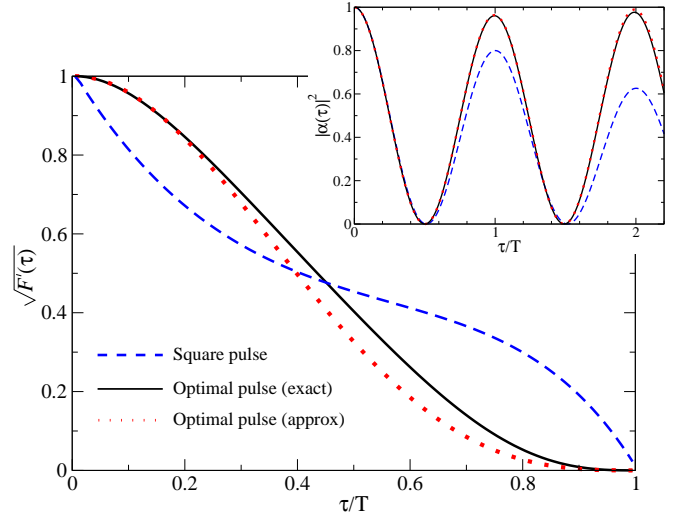


FIG. 3. Fidelity $F'(\tau)$ for subensemble selected by square preparation pulse (blue dashed line) and by optimal pulse of Eq. (14): exact solution (black solid line) and approximate analytical solution of Eq. (15) (red dotted line). The duration of the pulses is $T = 10\tau_{\text{tr}}$. Inset: Rabi oscillations of a single excitation between the field and the correspondingly selected subensemble.

in Fig. 3 and is given by

$$F'(\tau) \simeq \left| \frac{(T - \tau) \cos(\pi\tau/T)}{T} + \frac{\sin(\pi\tau/T)}{\pi} \right|^2 \approx 1 - \frac{\pi^2 \tau^2}{T^2}. \quad (15)$$

Hence, for short times $\tau \ll T$, the fidelity loss is quadratic in τ , which should be contrasted with linear fidelity loss for the square preparation pulse.

In the inset of Fig. 3, we show the Rabi oscillations of a single excitation between the field and the selected subensemble. Using the fidelity of Eq. (15), we find the following solution of Eq. (2) up to second order in τ_{tr}/T :

$$\alpha(\tau) \approx \left(1 - \frac{\tau_{\text{tr}}^2}{T^2} \right) \cos(\pi\tau/\tau_{\text{tr}}) + \frac{\tau_{\text{tr}}^2}{T^2}, \quad (16)$$

which yields $|\alpha(\tau_{\text{tr}})|^2 \approx 1 - 4(\tau_{\text{tr}}/T)^2$ and $|\alpha(2\tau_{\text{tr}})|^2 \approx 1 + \mathcal{O}(\tau_{\text{tr}}/T)^3$. Remarkably, the probability of excitation retrieval into the field mode is higher at the end of the second oscillation at $\tau = 2\tau_{\text{tr}}$ than the end of the first at $\tau = \tau_{\text{tr}}$. [Note that at the end of the second Rabi cycle the combined state of the field and SE has the initial phase, $|\Psi(2\tau_{\text{tr}})\rangle \simeq |1, \psi_0\rangle$.] In general, for this spectrum the retrieval infidelity at even revivals is 3rd order in τ_{tr}/T , as opposed to 2nd order at odd revivals.

(b) To maximize the fidelity at a specific storage time τ_s , we assume that the preparation time is a multiple of τ_s , $T = m\tau_s$ ($m \in \mathbb{Z}$). The solution of Eq. (12) is then

$$\Omega(t) = \Omega_0 \xi(t - \tau_s \lfloor t/\tau_s \rfloor) \sin \left(\pi \frac{\lfloor t/\tau_s \rfloor + 1}{m+1} \right),$$

where $\Omega_0 = \frac{\pi}{2} \tan \left(\frac{\pi/2}{m+1} \right)$, $\lfloor \dots \rfloor$ denotes the integer part (floor) of the expression, and $\xi(t)$ is a temporal profile

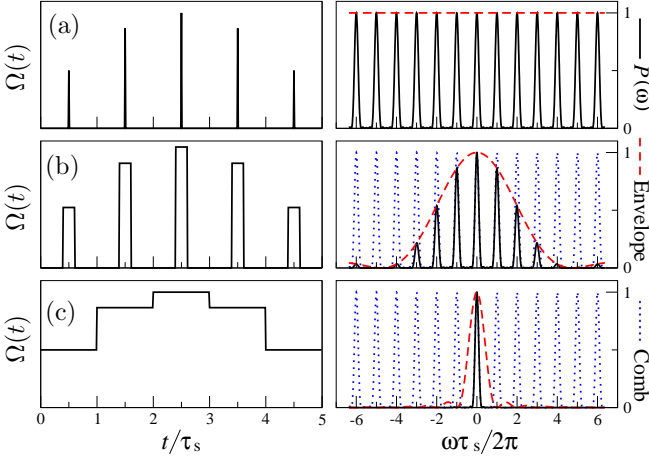


FIG. 4. Left column: preparation fields $\Omega(t)$ of Eq. (17), optimized for storage time τ_s , consisting of (a) series of instantaneous pulses $\xi(t) = \delta(t - \tau_s/2)$, (b) series of short pulses of width $\tau_s/5$, and (c) step function constant during each τ_s interval $\xi(t) = 1/\tau_s$. The preparation time is $T = 5\tau_s$. Right column: the resulting selection spectrum $P(\omega)$ of Eq. (18) given by the product of comb and envelope. All the spectra have the same fidelity $F'(\tau_s)$ but different number of spins $N' \propto \int_0^{\tau_s} \xi^2(t) dt$.

within τ_s of unity area $\int_0^{\tau_s} \xi(t) dt = 1$. Hence, the preparation pulse is an m -fold repetition of $\xi(t)$, at each step $0 \leq n < m$ multiplied by a different amplitude,

$$\Omega(t) = \Omega_0 \xi(t - n\tau_s) \sin\left(\pi \frac{n+1}{m+1}\right) \text{ for } t \in [n\tau_s, (n+1)\tau_s), \quad (17)$$

so that $A = \pi/2$ and $N' = \pi n_0(m+1)\Omega_0^2 \int_0^{\tau_s} \xi^2(t) dt$. The resulting approximate selection spectrum of Eq. (10),

$$P(\omega) \approx \left| \sum_{n=0}^{m-1} e^{-i\omega n\tau_s} \sin\left(\pi \frac{n+1}{m+1}\right) \right|^2 |\tilde{\xi}(\omega)|^2, \quad (18)$$

is a product of two terms: a “comb” term, which is a series of optimally shaped peaks spectrally separated by $2\pi/\tau_s$; and an “envelope” term, $\xi(\omega) \equiv \int_0^{\tau_s} e^{-i\omega t} \xi(t) dt$, wider than $2\pi/\tau_s$. Examples of $P(\omega)$ for various $\xi(t)$ are shown in Fig. 4. For a uniform $\xi(t) = 1/\tau_s$, $P(\omega)$ reduces to a single peak, since the envelope has a width of $2\pi/\tau_s$, resulting in the minimal number of selected spins $N' \propto 1/\tau_s$. By contrast, choosing $\xi(t)$ to be a narrow pulse of width $\delta t < \tau_s$ yields multiple ($\sim \tau_s/\delta t$) peaks within the envelope, and hence larger number of selected spins, $N' \propto 1/\delta t$. A delta function, $\xi(t) = \delta(t - \tau/2)$, will give rise to infinitely many peaks and thus an infinite number of spins $N' \rightarrow \infty$. In practice, however, the number of selected peaks is limited by the width of the original SE spectrum $n(\omega)$. The resulting fidelity [Eq. (11)] at time τ_s is now given by

$$F'(\tau_s) = \cos^2\left(\frac{\pi}{T/\tau_s + 1}\right). \quad (19)$$

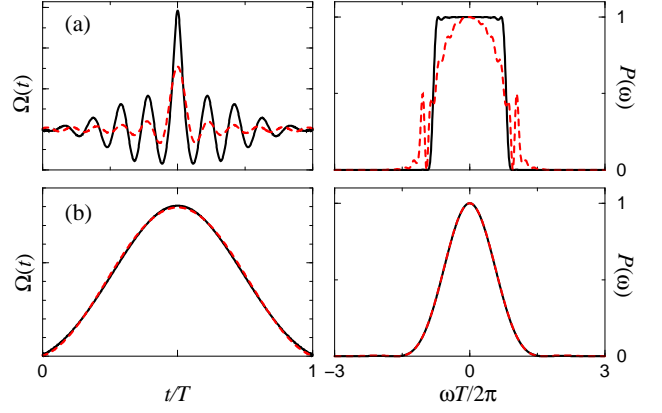


FIG. 5. Left column: preparation fields $\Omega(t)$ optimized for (a) long and (b) short preparation time T . In (a) the dashed (red) curve is the truncated sinc function, and the solid (black) curve is the numerically optimized pulse. In (b) the dashed (red) curve is the $\sin(t)$ function of Eq. (14) and the solid (black) curve is the numerically optimized pulse. Right column: the resulting exact selection spectra $P(\omega)$.

Note that the choice of $\xi(t)$ does not affect the fidelity [unlike the number of selected spins $N' \propto \int_0^{\tau_s} \xi^2(t) dt$], which is to be expected, since different solutions must, by definition, yield the same maximal fidelity.

B. Numeric solutions

We now solve the Euler-Lagrange equation (9) numerically using the exact selection spectrum of Eq. (7). We maximize the subensemble fidelity $F'(\tau_s)$ for two distinct situations:

a. Stable spins – In this scenario, we can use a long preparation time T so as to achieve the optimal rectangular spectrum $P(\omega)$ of the subensemble. Our main constraint is the number of selected spins N' . Clearly, for infinite T the preparation pulse $\Omega(t)$ is a sinc function; for long but finite T , however, the optimal pulse is modified but still resembles the sinc function, as shown in Fig. 5(a).

b. Short-lived spins – Now the short lifetime of the spins severely limits the preparation time T . The resulting spectral width of the selected subensemble $\Delta \sim 2\pi/T$ will be wide enough to contain many spins. We can therefore disregard the constraint on N' , focusing instead on achieving the narrowest possible selection spectrum $P(\omega)$ within a given preparation time T . We then find that the optimal pulse $\Omega(t)$ is almost identical to that in Eq. (14), see Fig. 5(b).

C. Fidelity loss during the QI transfer and storage

The preparation selects a subensemble of $N' \ll N$ spins with reduced spectral variance $\langle(\omega - \omega_0)^2\rangle \simeq$

$(\frac{2}{\pi})^8 (N'/n_0)^2$, but also results in a longer transfer time $\tau_{\text{tr}} = \pi/\bar{\eta}\sqrt{N'}$. During the transfer, the fidelity loss, or error due to the decay κ of the (cavity) field is $\epsilon_{\text{tr}} \sim \kappa\tau_{\text{tr}}$, while the error accumulated during the storage time τ_s in the subensemble is $\epsilon_s \sim \langle(\omega - \omega_0)^2\rangle\tau_s^2$. Minimizing the total error $\epsilon = \epsilon_{\text{tr}} + \epsilon_s$ with respect to N' , we obtain

$$\min(\epsilon) \sim 2 \left(\frac{\kappa^2 \tau_s}{\bar{\eta}^2 n_0} \right)^{2/5} \quad \text{for } N' = \frac{\pi^{18/5}}{2^4} \left(\frac{\kappa n_0^2}{\bar{\eta} \tau_s^2} \right)^{2/5}. \quad (20)$$

Hence, small error ϵ requires large cooperativity $\bar{\eta}^2 n_0 \gg \kappa^2 \tau_s$ of the field-subensemble coupling.

V. EXPERIMENTAL CONSIDERATIONS

To illustrate the results of the foregoing discussion, we consider a SE of NV color centers in diamond coupled to a superconducting coplanar waveguide resonator [1–5]. The ground $|g\rangle$, excited $|e\rangle$ and auxiliary $|a\rangle$ states correspond, respectively, to the $m = 0$, $m = 1$ and $m = -1$ Zeeman sublevels of the ground electronic (spin-triplet) state of the NV. Transitions $|g\rangle \rightarrow |e\rangle$, $|a\rangle$ have the frequencies of around 2.88 GHz, and can be selectively addressed by the external σ_{\pm} -polarized microwave fields. In addition, a static magnetic field can be used to tune the transition $|g\rangle \rightarrow |e\rangle$ in and out of resonance with the cavity mode ω_0 . The experimental inhomogeneous spectrum of the ensemble of $N \approx 10^{12}$ NV centers [5] has the total width of about $\Delta/2\pi \sim 7$ MHz, composed of three partially overlapping Lorentzians of widths ~ 2.6 MHz split by ~ 2.2 MHz due to the hyperfine coupling to the $I = 1$ nuclear spin of the ^{14}N atom. This results in a storage fidelity of $F(\tau_s) \approx 1 - \tau_s/60$ ns, while with the collective SE-cavity coupling strength $\bar{\eta}\sqrt{N} \simeq 2\pi \times 13$ MHz [5], the excitation transfer time is $\tau_{\text{tr}} \approx 40$ ns.

As an example, assume that one can achieve a cavity quality factor of $Q = 10^6$. The photon lifetime in the cavity, $\kappa^{-1} \approx 55$ μs , is then much longer than the transfer time τ_{tr} . This allows a preparation that reduces the ensemble spectral width Δ , and the number of spins N , by a factor of $5 \cdot 10^3$ —almost 4 orders of magnitude—while still keeping the transfer time $\tau_{\text{tr}} \approx 2.8$ $\mu\text{s} = 0.05\kappa^{-1} \ll \kappa^{-1}$ well within the cavity lifetime. The new subensemble, created by the optimal preparation pulse $\Omega(t)$ of $T = 0.7$ ms duration, has the storage fidelity $F'(\tau_s) \approx 1 - (\tau_s/0.22 \text{ ms})^2$. There is negligible loss of fidelity during the QI transfer from the cavity field to the new ensemble, which can now store QI for $\tau_s \simeq 50$ μs with 95% fidelity, compared to 3 ns with the original SE and 2.8 μs inside the cavity.

Had we used the square preparation pulse of the same duration, the new ensemble would have had a storage

fidelity of $F'(\tau_s) \approx 1 - \tau_s/0.17$ ms, which can store QI for $\tau_s \simeq 8.5$ μs with 95% fidelity—more than 5 times worse than with the optimal preparation.

VI. SUMMARY AND DISCUSSION

Ensembles of long-lived two-level systems—spins—can serve as collective quantum memories, but their utility is often compromised by the inhomogeneous spectral broadening, which can hamper both the QI transfer to and from the SE and reduce the storage fidelity. The fidelity is very sensitive to not only the width Δ but also to the profile of the inhomogeneous spectrum of the SE. Specifically, for a spectrum with long wings, such as a Lorentzian, the loss of fidelity during the storage time $\tau_s \ll \Delta^{-1}$ scales linearly with $\Delta\tau_s$, while for a spectrum with sharp cutoff at Δ the loss of fidelity is quadratic in $\Delta\tau_s$.

In this paper, we have proposed and analyzed a method to select a spectrally narrow subensemble of spins, which maximizes the fidelity of QI storage and also improves the QI transfer from and to an electromagnetic field. In our method, judiciously designed pulses of finite duration—determined by the spin relaxation time—select the optimal spectral portion of a large SE, while the remaining spins making up the spurious part of the spectrum are discarded by transferring them to an auxiliary metastable state. Our method is applicable to microwave cavity (circuit QED) based hybrid quantum systems involving QI processing qubits and SE quantum memories, as well as to optical field storage in SEs. In the former case, the QI transfer time is limited by the cavity field relaxation time, while in the latter case, it is the interaction (transit) time of the optical pulse with the SE. Hence, one always has to attain large cooperativity in ensemble-field coupling by having many spins, and large optical depth.

Similar considerations may apply to the schemes involving noisy processing qubits coupled directly to the spin-ensemble memories, such as, e.g., the electron spin of a NV center interacting with the surrounding ensemble of long-lived nuclear spins [44–49]. Then in Eq. (20) one would have to replace κ by the qubit decoherence rate γ . As opposed to the cavity decay rate κ , which is difficult to change, γ of a qubit can be suppressed by dynamical control methods [50–55], which, for a fixed error ϵ , will result in quadratic increase of the memory time $\tau_s \propto \epsilon^{5/2}/\gamma^2$.

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